3.2 Perfect Squares, Perfect Cubes, and Their Roots

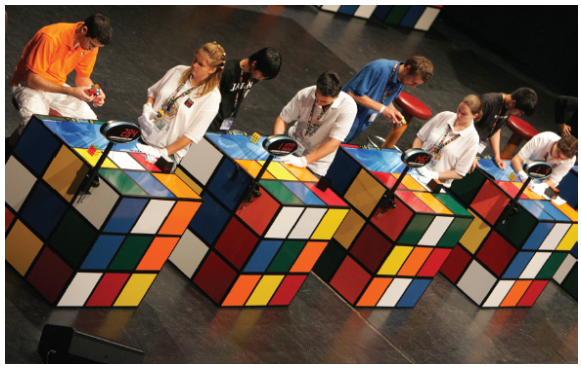
LESSON FOCUS Identify perfect squares and perfect cubes, then determine square roots and cube roots.

Make Connections

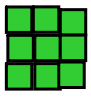
The edge length of the Rubik's cube is 3 units.

What is the area of one face of the cube?
Why is this number a *perfect square*?

What is the volume of the cube?
This number is called a **perfect cube**.
Why do you think it has this name?



Model all the perfect squares from 1 to 100.
Record the corresponding perfect squares.



1, 4, 9, 16

1x1 2x2 3x3

Model all the perfect cubes from 1 to 100.
Record the corresponding perfect cubes.

$V = l \cdot w \cdot h$

1, 8, 27

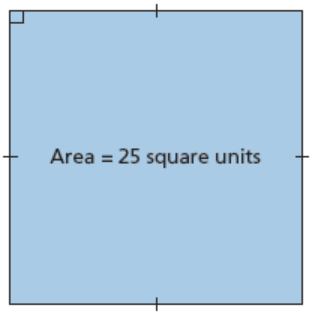
$1 \times 1 \times 1$ $2 \times 2 \times 2$ $3 \times 3 \times 3$

3.2 Perfect Squares, Perfect Cubes, and Their Roots

Any whole number that can be represented as the area of a square with a whole number side length is a *perfect square*. The side length of the square is the *square root* of the area of the square.

We write: $\sqrt{25} = 5$

25 is a perfect square and 5 is its square root.



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3.2 Perfect Squares, Perfect Cubes, and Their Roots

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Any whole number that can be represented as the area of a square with a whole number side length is a perfect square.

$A = l \cdot w$

Any whole number that can be represented as the volume of a cube with a whole number side length is a perfect cube.

$V = l \cdot w \cdot h$

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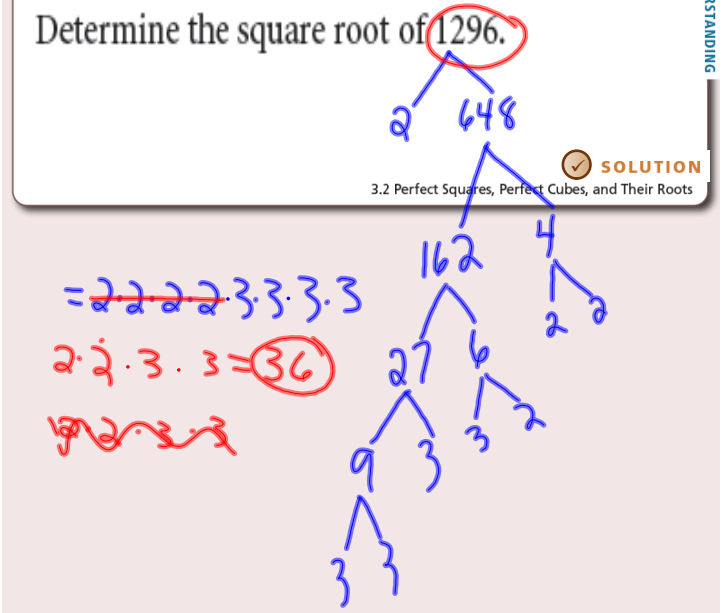
Example 1 Determining the Square Root of a Whole Number

One possible method:
Step One: Find the prime factors
Step Two: Separate the factors into two equal groups
Step Three: Find the product of one group of primes.

Determine the square root of 1296.



SOLUTION

CHECK YOUR UNDERSTANDING

Example 1 Determining the Square Root of a Whole Number

Determine the square root of 1296.

SOLUTIONS

Method 1

Write 1296 as a product of its prime factors.

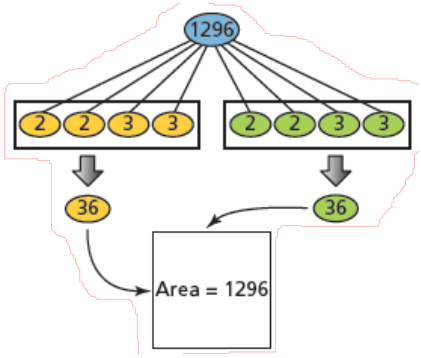
$1296 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ Group the factors in pairs.

$= (2 \cdot 2)(2 \cdot 2)(3 \cdot 3)(3 \cdot 3)$

$= (2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3)$ Rearrange the factors in 2 equal groups.

$= 36 \cdot 36$

Since 1296 is the product of two equal factors, it can be represented as the area of a square.



So, the square root of 1296 is 36.

(Solution continues.)

Example 1 Determining the Square Root of a Whole Number

Method 2

Estimate:

$30^2 = 900$ and $40^2 = 1600$

$900 < 1296 < 1600$

So, $30 < \sqrt{1296} < 40$

1296 is about halfway between 900 and 1600.

So, $\sqrt{1296}$ is about halfway between 30 and 40.

Use guess and test to refine the estimate.

Try 35: $35^2 = 1225$ (too small, but close)

Try 36: $36^2 = 1296$

So, the square root of 1296 is 36.

CHECK YOUR UNDERSTANDING

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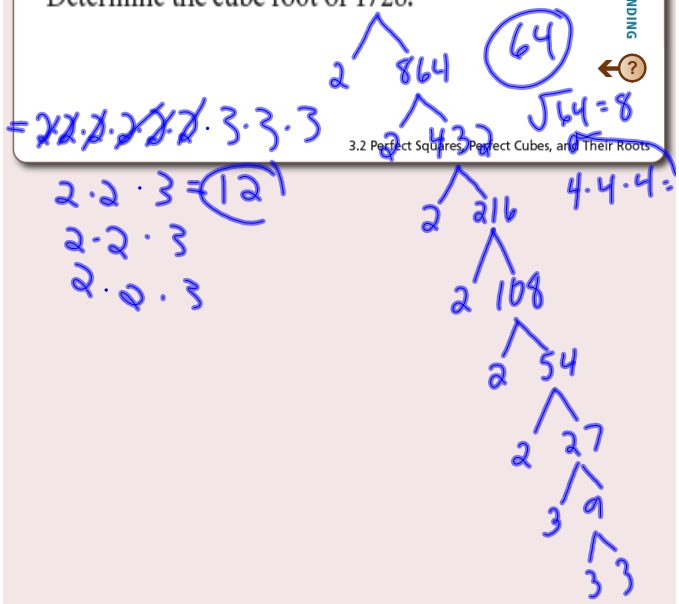
Example 2 Determining the Cube Root of a Whole Number

SOLUTION

One possible method:
Step One: Find the prime factors
Step Two: Separate the factors into three equal groups
Step Three: Find the product of one group of primes.

CHECK YOUR UNDERSTANDING

Determine the cube root of 1728.



Example 2 Determining the Cube Root of a Whole Number

Determine the cube root of 1728.

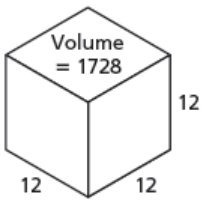
SOLUTIONS

Method 1

Write 1728 as a product of its prime factors.

$1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$ Group the factors in sets of 3.
$= (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(3 \cdot 3 \cdot 3)$ Rearrange the factors in 3 equal groups.
$= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$
$= 12 \cdot 12 \cdot 12$

Since 1728 is the product of three equal factors, it can be represented as the volume of a cube.



So, the cube root of 1728 is 12.

(Solution continues.)

Example 2 Determining the Cube Root of a Whole Number

Method 2

Estimate:
$10^3 = 1000$ and $20^3 = 8000$

$1000 < 1728 < 8000$

So, $10 < \sqrt[3]{1728} < 20$

1728 is closer to 1000 than to 8000.

So, $\sqrt[3]{1728}$ is closer to 10 than to 20.

Use guess and test to refine the estimate.
Try 11: $11^3 = 1331$ (too small, but close)
Try 12: $12^3 = 1728$

So, the cube root of 1728 is 12.

CHECK YOUR UNDERSTANDING

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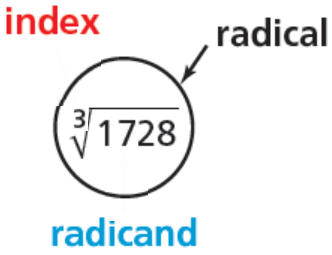


The index number tells you what root you are finding (usually square or cubed root). If there isn't an index number visible it is assumed that it is an index of 2.

Example 3 Using Roots to Solve a Problem

A cube has volume 4913 cubic inches. What is the surface area of the cube?

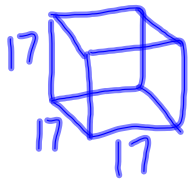
SOLUTION

$V = 4913$, $SA = ?$

$17 \quad 289$

$17 \quad 17$

$\sqrt[3]{4913} = 17$



$17 \times 17 = 289 \times 6 =$

1734 inches^2

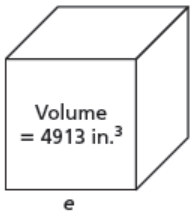
CHECK YOUR UNDERSTANDING

Example 3 Using Roots to Solve a Problem

A cube has volume 4913 cubic inches. What is the surface area of the cube?

SOLUTION

Sketch a diagram.



To calculate the surface area, first determine the edge length of the cube.

The edge length e, of a cube is equal to the cube root of its volume.

$e = \sqrt[3]{4913}$

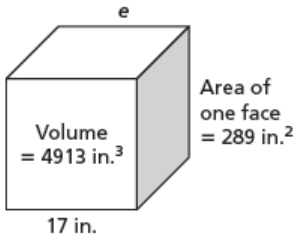
$e = 17$

The surface area, SA, of a cube is the sum of the areas of its 6 congruent square faces.

$SA = 6(17 \cdot 17)$

$SA = 6(289)$

$SA = 1734$



The surface area of the cube is 1734 square inches.

CHECK YOUR UNDERSTANDING

3.2 Perfect Squares, Perfect Cubes, and Their Roots

Classwork/Homework

Pg.146 #4a, 5a, 6ab, 7a, 8a, 10

You have the rest of class to work on these questions. Please make sure to finish these questions for homework if necessary.